BOOK REVIEW

A History of Pi, by Petr Beckmann. St. Martins's Press, 1976, 200 pp., $11.95; Barnes and Noble Books, 1989, $14.95.
The Joy of Pi, by David Blatner. Walker & Co., 1997, 144 pp., $18.00.
The Nothing That Is, by Robert Kaplan. Oxford University Press, 1999, 225 pp., $22.00.
e: **The Story of a Number**, by Eli Maor, Princeton University Press, 1998, 232 pp., $14.95.
An Imaginary Tale, by Paul Nahin, Princeton University Press, 1998, 248 pp., $24.95.
Zero: The Biography of a Dangerous Idea, by Charles Seife. Viking Press, 2000, 248 pp., $24.95.

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We are in the midst of an onslaught of books devoted to particular numbers. In this article we will be concerned with several recent and not-so-recent books about π, e, i, and 0; of the quantities in the famous equation $e^{\pi i} + 1 = 0$, only 1 does not yet have its book. Given the trend, we can look forward to its appearance, possibly followed by volumes on "+" and "=".

The practice of devoting a book to a particular number is by no means a new development. In 1913 Ernest William Hobson, a distinguished mathematician and expositor, wrote a brief but excellent account of π [**12**]. Featuring serious history and serious mathematics, Hobson's *Squaring the Circle*, elementary though it is, inevitably became forbidding as a popular reference. By 1970 the time was ripe for a book on π that did not have quite so stiff an upper lip. Who better to write such a book than an electrical engineer?

Freed from the "dispassionate aloofness of the historian and the tiresome rigor of the mathematician," Petr Beckmann fashioned *A History of Pi* for a new age. Light and breezy, it is the kind of book that you can devour during a short-hop flight. True to his word, Beckmann does not "complicate" his explanations by "excessive mathematical rigor." Nevertheless, there is just enough mathematical detail to keep the book respectable. In particular, Beckmann provides a first rate (but no longer up-to-date) glimpse into the methods of the digit hunters. There can be no question that Beckmann struck an agreeable balance. His book has been deservedly successful, remaining in print for more than thirty years.

Of course, just as there was a shift in paradigm between 1913 and 1970, so has there been a shift between 1970 and the present. Nowadays who is better qualified to write a book on π than an expert in computer publishing, especially if he has already written books on digital imaging and virtual reality? Enter David Blatner and *The Joy of Pi*. When I saw this book's price listed as $18.00(5.73$\pi$) on the inside cover flap, I was prepared to offer it a hearty welcome. Alas, a reviewer is not allowed to judge a book by its cover: as I turned over the first page, disillusionment set in. The back cover blurb trumpets that *The Joy of Pi* is "beautifully designed" and "whimsically formatted" with a "creative graphic layout." Indicative of that creative layout is an interesting straightedge and compass diagram. It *seems* to be an exquisite historical example of the circle-squarer's art, but that is just my guess: although the figure appears ten times throughout the book it is described nowhere.

There are many other instances where the book could stand to be less whimsically formatted. Not one of the formulas is labeled or even referred to. On page 71, for example, there appears a truly remarkable infinite series for π. Since it is given in the chapter on the Chudnovsky brothers, the reader may suspect that it is their discovery and indeed it is. But surely the reader should not have to guess. The author tells us that the computation of 707 digits of π in 1873 "was hailed throughout the civilized world as the unveiling of a great mathematical truth," but he does not inform us that each additional term of the Chudnovskys' series yields an additional 14 digits of π. It is a telling oversight.

Whereas Hobson expected his readers to stick with him as he led them through a demonstration of the transcendence of π, Beckmann, who attempted nothing nearly as arduous, found it necessary to advise his readers to skip over the mathematics that they found too difficult. By contrast, *The Joy of Pi* resembles grade school science texts that have been shorn of all meaningful content so as to furrow no brow. Although we learn, for example, that Snell "believed firmly in his theories [but] he was never able to prove them," we do not learn exactly what those theories were.

The Joy of Pi is amusing in places and has some trivia that readers might find curious. Nevertheless, I would advise those readers of this journal who want to learn more about π and its history to turn to Beckmann supplemented by [**3**], [**6**], [**12**], and [**16**]. Having done so they will find Blatner's book utterly superfluous. Indeed, since π has been so well taken care of, we may safely turn our attention to other numbers: the number *e*, for example.

In retrospect, a book on *e* was a natural. With the longevity of *A History of Pi* so plainly in view, one wonders that so many years passed before such a book appeared. Patterned on Beckmann's successful model, Eli Maor's *e: The Story of a Number* rolled off the presses in 1994. A very slightly augmented paperback printing was released in 1998.

Persuading readers that the number *e* is a natural subject for study is no easy task, but historical exposition is perfect for the job. Unfortunately, Maor is not entirely convincing when he discusses the origins of *e*. In his preface Maor suggests that *e* first appeared in connection with the formula for compound interest, an assertion he repeats in his third chapter. Is there any evidence for his conclusion? The leap from daily compounding to continuous compounding would have been astounding at the time the number *e* was introduced. Yet not even one case of any type of subannual compounding is documented in Maor's book. Furthermore, although records of compound interest date back to antiquity, the appearance of *e* was exactly contemporaneous with the introduction of the Napierian logarithm. How can Maor's hypothesis be reconciled with the chronology?

Exacerbating these problems, Maor's analysis of Napier's contribution is confusing. He writes that Napier chose "for a base a number small enough so that its powers will grow reasonably slowly." The entire discussion is predicated on the notion of a base. Yet Maor writes in a footnote to this very discussion, "As we have seen, he [Napier] did not think in terms of a base, a concept that developed only later." We have a sure guide to Napier's discovery of logarithms: Napier's own account of his thinking. That report will leave the reader in awe. Not only does it reveal the role of *e* as natural base, it leaves us in admiration for its author, an amateur mathematician who hit upon an essential kinematical consideration of calculus even before the creation of the coordinate plane and analytic geometry. Of course, few will find it pleasurable to read a technical work written in archaic language. Maor provides a sketchy reconstruction of Napier's method that he hides in an appendix. More thorough accounts of Napier's construction of logarithms may be found in [**1**], [**5**], and [**9**].

Several historical nits can be picked. Maor writes that Johannes Kepler formulated his three laws of planetary motion in Germany. In fact, during Kepler's lifetime there was no state that could be referred to as "Germany." Even if there were, Kepler derived and formulated his laws during the twenty-six years he worked in Prague and Linz. Referring to the twentieth century discovery of *The Method of Archimedes*, Maor asserts that "J. L. Heiberg found a medieval manuscript in Constantinople." In fact, Heiberg identified that manuscript and others. He was actually in Copenhagen when he came across a catalogue description of a then unidentified palimpsest (containing *The Method* and several other manuscripts) in Constantinople. These inaccuracies concern facts that are unimportant to the author's story but they are not trifles, as I will argue later.

As is to be expected, the mathematical portion of Maor's tale is generally sound. In one uncharacteristic lapse, Maor illustrates the solution of the Brachistochrone Problem with a sketch of a cycloid generated by a circle rolling on top of the x-axis with the positive y-axis pointing upward. For a correct diagram accompanied by a proof, as well as mathematical treatments of many topics that Maor only talks about, the delightful book of Simmons, *Calculus Gems*, is recommended [**18**]. Indeed, *Calculus Gems* and the supplemental references [**1**], [**5**], and [**9**] remain my favorite resources for the topics that Maor covers. However, I am sure that others may well find Maor's book an attractive and convenient choice.

With histories of π and e already in the bag, you can confidently predict what was bound to come along next. Sure enough, Paul Nahin's *An Imaginary Tale: The Story of* $\sqrt{-1}$ made its appearance in 1998. Like Beckmann, Nahin is a professor of electrical engineering. Unlike either Beckmann or Maor, Nahin had the ambitious plan of making mathematical computation an integral part of his narrative. He is certainly to be applauded for his concept. I will not say much about the execution of the concept. For one thing, *An Imaginary Tale* has already received a thorough review [**2**]—I wrote it myself! Writing for the professional mathematician I felt obliged to expose a litany of historical, mathematical, and pedagogical problems. There is much to like about *An Imaginary Tale* but the bottom line is that it has too many serious flaws to merit a general recommendation.

Mathematics is filled with curious and interesting numbers. Taken together, with a paragraph or two devoted to each, they constitute the basis for a charming and interesting book [**20**]. The problem is, how do you jump on the bandwagon and build an entire book around a single number? Take the number 0, for instance. What would you have to say about it? Well, we now know what Robert Kaplan, a retired high school teacher, and Charles Seife, a science journalist, would say and we are scarcely the wiser for it. Robert Kaplan's *The Nothing That Is: A Natural History of Zero* arrived from Oxford University Press late in 1999. Its appearance must have been mortifying for Charles Seife, whose own *Zero: The Biography of a Dangerous Idea*, was scheduled for release (or rushed to press?) in February 2000.

Not surprisingly, the two books have much in common. Each book fleshes out its slender premise with the same sort of slightly off-topic discussion. They share, for example, similar treatments of infinitesimals and the development of the differential calculus. Both books are written informally with an individual style not usually found in mathematics books. And both books are chock-full of misconceptions, half-truths, outright lies, and mumbo-jumbo. Consider this composite passage for a brief taste of their flavor: "If you look at zero you see nothing; but look through it and you will see the world. It provides a glimpse of the ineffable and the infinite. Zero shaped humanity's view of the universe—and of God." The numerous similarities notwithstanding, the two books are not equivalent. In fact, one is merely mediocre whereas the other is truly vile.

Let us dispose of the latter with the candor it deserves. Charles Seife's intent is to sensationalize at every turn: if the truth suffers, then so be it. He begins his story with a rousing bait-and-switch: "Zero hit the USS Yorktown like a torpedo . . . Though it was armored against weapons nobody had thought to defend the Yorktown from zero . . . No other number can do such damage." With these words Seife is alluding to a division by zero error that crashed the computers that controlled the missile cruiser's engines. Although the Yorktown incident is of vital interest for several reasons, division by zero is not one of them. Go ahead—divide by zero on your calculator. Did you break it? As to Seife's assertion that no other number can do such damage, I leave the refutation as an exercise with a hint: an equally dangerous floating point exception can be found in the title of one of the books I have discussed.

Is Seife guilty of nothing more than journalistic exuberance? No! According to Seife, "every time mathematicians tried to deal with the infinite or with zero they encountered trouble with illogic." Seife tells his readers: "If you were to throw a dart at the number line it would never hit a rational number. Never." He asserts that the series $1 - 1 + 1 - 1 + 1 - \cdots$ "can equal 0 and 1 at the same time." Ironically, Seife repeats a favorite old myth, passed down from one popularizer to the next, that Euler was easily bamboozled by divergent series. "The careless manipulation of zero and infinity led him astray," says Seife. Popularizers hold onto their cherished fables as tenaciously as the circle-squarers hold onto their delusions. You cannot tell them otherwise. On the matter of divergent series Hardy tried more than fifty years ago [**10**]: "It is a mistake to think of Euler as a 'loose' mathematician . . . his language sometimes suggests a point of view far in advance of the general ideas of his time. Here as elsewhere, Euler was substantially right." There are many other examples of the popularizer's knack for confabulating the historical record. Seife joins Maor in propagating the myth of the Greek *horror infiniti*. Read [**11**] as an antidote.

Why does Seife adopt a snide, smirky, spiteful tone for his book? Why must he refer to Martin Luther as a "constipated German monk"? He describes Kronecker as "the mathematician who would hound Cantor into a mental institution." This is a serious charge that is based on nothing more than the fictional writing of E.T. Bell. I could suggest [**8**] for a well-researched account but that would only be a start: there is not enough space in this review to set right all that is wrong with Seife's book.

Turning to Robert Kaplan's book is something of a relief. In his hands the story of zero is initially made to seem worth the telling. For the first eight chapters I was not tempted to conclude that Kaplan had filled a much-needed gap in the literature (to borrow a quip from classics professor Moses Hadas). Right from the start, however, I found Kaplan's florid prose to be an obstacle. The phrase "recursive abstracting" is listed five times in the index. I am still not sure what it means but Kaplan assures us that it "is the very stuff of mathematics." To make the matter clearer he adds that it is "this abbreviating the sweep of landscape you have just taken in to an aperçu for a higher order of seeing."

Language and style may be a matter of taste but facts are facts. Referring to a mathematician whose name is associated with a well-known limit theorem of calculus, Kaplan asks "Shall we call him Guillaume François Antoine, Marquis de l'Hôpital?" No we shan't! Guillaume de l'Hôpital was the Marquis de Sainte-Mesme. In a similar vein, Gauss's first name is spelled as "Karl" instead of "Carl." Mathematical facts are also garbled. Kaplan describes Ramsey Theory as "a branch of mathematics that studies rapidly growing functions." Later Kaplan gives a construction of what he calls the Farey sequence but which is in reality the Stern-Brocot sequence. The oft-repeated "history" of Farey sequences that Kaplan resurrects was debunked five years ago when Bruckheimer and Arcavi consulted the primary sources [**4**].

Given that Ramsey Theory, Farey sequences, and the names of mathematicians have little to do with the history of zero, it is reasonable to wonder if any of this matters. Let me be clear on this issue: these errors are of the utmost importance. In mathematics we read an author's assertion and his proof of it; in doing so we are able to ascertain validity. That is not how we as mathematicians receive history. If we cannot rely on the say-so of a historian, then his writing is worthless to us. In Kaplan's case we have grounds to question his understanding of mathematics and reason to reject his standards of accuracy. In short, why should we be convinced by what he has to say? Let me illustrate.

One of the key historical problems concerning zero is the extent of the influence that the ancient Babylonian zero had on the Hindu zero. This is a question that the experts have batted around for some time without coming to agreement amongst themselves. Ifrah, for example, concludes that India came upon zero on its own [**13**, p. 341]. Freudenthal, on the other hand, has argued that the Hindu zero originated with the Babylonians. After presenting Freudenthal's arguments, Van der Waerden [**19**, p. 57] will allow no more than that Freudenthal's scenario "is quite possible." Menninger [**14**, p. 399] concurs that the events of Freudenthal's theory could have happened "and with this 'could have' we shall have to be satisfied." Now Kaplan, who comes out unwafflingly for Feudenthal's hypothesis, dismisses the opposing point of view with nothing more than these words "these disputes ... are paved wall to wall with fallacies of negative, presumptive and possible proofs, fallacies pragmatic and fallacies aesthetic." Perhaps, but based on Kaplan's track record why should the reader accept his authority?

Although *The Nothing That Is* has been decorated with attractive art drawn by the author's wife, nobody thought to include any maps. One of the most important "documents" in the history of zero is a tablet that was unearthed at Kish. Would the reader not want to know that this ancient city, long since vanished, was situated at the narrowing of the gap between the Tigris and Euphrates rivers, south of present-day Baghdad. Avicenna is said to have been born in Bukhara. Will that be meaningful to readers outside of Uzbekistan? Not even an atlas will help locate the many Arab mathematicians who are cited but for whom no geographic information is given.

Although I would not steer the general reader with a casual interest in the history of numbers away from Kaplan's book, I would recommend that he try alternatives such as [**7**] first. Readers of this journal who are interested in the development of the concept of zero should *bypass The Nothing That Is* and seek out works of greater scholarship ([**13**], [**14**], [**15**], and [**19**]).

There! I let slip the word "scholarship." There was a time when that was the mission of the university presses. They were the publishers of last resort for serious work of scholarly value but specialist appeal. Compare the books of original research that the university presses used to publish ([**12**], [**14**], [**15**], [**19**]) with the recent crop of popular rehashings. Nowadays it would seem that the harvesting of jacket fodder has assumed priority over even routine copy editing. Princeton University Press, for example, boasts of Nahin's literary accomplishments on the outside cover of his book, yet on the inside we are treated to the following not atypical sentence (p. 82): "In fact, Gauss had been in possession of these concepts in 1796 (before Wessel) and had used them to reproduce, without Gauss' [sic] knowledge, Wessel's results." When Princeton reprinted Maor's book in 1998, four pages concerning a new Mersenne prime were added. Could the footnote referring to the "unpublished" proof of Fermat's Last Theorem not have been updated? Despite many reprintings since 1974, Beckmann's book still refers to the "unproven four color conjecture."

All of this is as nothing compared to the assault on scholarship spearheaded by Oxford University Press. You will search in vain for notes or a bibliography in Kaplan's book. That is because Oxford entirely removed these off-putting accouterments of scholarship, electing to post them on a website. It is ironic that Kaplan (p. 7) chortles that Sumerian clay tablets have outlasted computer punchcards of the 1960s. Of what value will his notes and bibliography be in five years when the "portable document format" (or pdf) has yielded to the next improvement in electronic archiving? Of what value will his documentation be in ten years when Oxford finds it unprofitable to waste disk space on a file pertaining to an out-of-print book? For the time being, no one should be deceived into thinking that the web is a place for anything but ephemera. And no one should be deceived into thinking that the imprimatur of a leading university conveys authority and scholarship.

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