M233 Spring 2005 Homework 3

Due: 21 October 2005

In each exercise, $f(x, y) = x \ln(xy)$ and $(x_0, y_0) = (1, e)$.

1. Calculate $(\partial f/\partial x)(x_0, y_0)$ and $(\partial f/\partial y)(x_0, y_0)$. Also calculate the four second order partial derivatives of f at (x_0, y_0), verifying that $f_{xy}(x_0, y_0) = f_{yx}(x_0, y_0)$.

2. Using (x_0, y_0) as the base point, calculate the linear approximation of $f(e/3, 3)$.

3. The plane $y = e$ intersects the graph of $z = f(x, y)$ in a curve. Parameterize the tangent line to the curve at the point $(1, e, 1)$. The plane $x = 1$ intersects the graph of $z = x \ln(xy)$ in another curve. Parameterize the tangent line to this curve at the point $(1, e, 1)$.

4. Find an equation for the tangent plane to the graph of $z = f(x, y)$ at $(1, e, 1)$.

5. Suppose that
$$x = s^3 e^{st},$$
$$y = st^2 + e^{1+t}$$
and
$$z = f(x, y).$$
These three equations determine a function $z(s, t)$. Use the Chain Rule to calculate
$$\frac{\partial z}{\partial s}(1, 0) \quad \text{and} \quad \frac{\partial z}{\partial t}(1, 0).$$