

# Calculus Single Variable

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## Section 7.1 Integration by Parts

### **1. The Formula for Integration by Parts**

If  $u$  and  $v$  are functions, then

$$\frac{d}{dx}(u(x)v(x)) = \left(\frac{d}{dx}u(x)\right)v(x) + u(x)\left(\frac{d}{dx}v(x)\right).$$

As a result

$$\int \frac{d}{dx}(u(x)v(x)) dx = \int \left(\frac{d}{dx}u(x)\right)v(x) dx + \int u(x)\left(\frac{d}{dx}v(x)\right) dx,$$

or

$$u(x)v(x) = \int \left(\frac{d}{dx}u(x)\right)v(x) dx + \int u(x)\left(\frac{d}{dx}v(x)\right) dx,$$

or

$$\int u(x)\left(\frac{d}{dx}v(x)\right) dx = u(x)v(x) - \int \left(\frac{d}{dx}u(x)\right)v(x) dx.$$

It is convenient to remember the shorthand version of this equation:

$$\int u \, dv = u \, v - \int v \, du .$$

Notice that integration by parts does not by itself result in the evaluation of an integral. Instead, integration by parts transforms one integral, namely  $\int u \, dv$ , into a new integral, namely  $\int v \, du$ . In general, it is desirable that the new integral be easier than the original integral.

## 2. Strategy for Using Integration by Parts

In general, we think of using integration by parts when we see a product in the integrand. However, when we decide to try integration by parts on the integral

$$\int f(x) \, g(x) \, dx$$

we must decide which of  $f(x)$  and  $g(x)$  to set equal to  $u$ . (The other factor together with  $dx$  then becomes  $dv$ .)

The guiding principle is that in transforming  $\int u \, dv$  into the new integral  $\int v \, du$ , we differentiate  $u$  to obtain  $du$  and integrate  $dv$  to obtain  $v$ . We generally choose  $u$  and  $dv$  so that at least one of these operations results in a simpler expression. For example, when  $\ln(x)$  appears in an integrand, we usually set  $u = \ln(x)$  because then  $du = \frac{1}{x} \, dx$  and the algebraic term  $\frac{1}{x}$  is much simpler than the original  $\ln(x)$ . The same is true for expressions such as  $\arcsin(x)$

and  $\arctan(x)$ : if  $u = \arcsin(x)$  then  $du = \frac{1}{\sqrt{1-x^2}} \, dx$  and if  $u = \arctan(x)$  then  $du = \frac{1}{1+x^2} \, dx$ .

If the integrand contains  $x^n$  but not  $\ln(x)$ ,  $\arcsin(x)$ , or  $\arctan(x)$ , then we generally set  $u = x^n$ . The reason is that

$du = n x^{(n-1)} \, dx$  is simpler than the original term  $x^n$ , the power having been decreased by 1.

Since the expressions  $\cos(ax)$ ,  $\sin(ax)$ , and  $e^{(ax)}$  neither become neither simpler nor more complicated after differentiation or integration, they are usually not factors in our consideration of how to divide  $f(x) \, g(x) \, dx$  into  $u$  and  $dv$ .

## 2. Basic Examples

**Example**

Calculate  $\int x \cos(ax) dx$  for  $a \neq 0$ .

**Solution**

Set  $u = x$ ,  $du = dx$  and  $dv = \cos(ax) dx$ ,  $v = \frac{1}{a} \sin(ax)$ . Then

$$\int x \cos(ax) dx = \frac{1}{a} x \sin(ax) - \frac{1}{a} \int \sin(ax) dx = \frac{1}{a} x \sin(ax) + \frac{1}{a^2} \cos(ax) + C.$$

Verification:

```
[ > testeq(diff(x*sin(a*x)/a+cos(a*x)/a^2, x) = x*cos(a*x));
      true
```

**Example**

Calculate  $\int x e^{(ax)} dx$  for  $a \neq 0$ .

**Solution**

Set  $u = x$ ,  $du = dx$  and  $dv = e^{(ax)} dx$ ,  $v = \frac{1}{a} e^{(ax)}$ . Then

$$\int x e^{(ax)} dx = \frac{1}{a} x e^{(ax)} - \frac{1}{a} \int e^{(ax)} dx = \frac{1}{a} x e^{(ax)} - \frac{1}{a^2} e^{(ax)} + C.$$

Verification:

```
[ > testeq(diff(x*exp(a*x)/a-exp(a*x)/a^2, x) = x*exp(a*x));
      true
```

## 4. Antiderivatives of $\ln(x)$ , $\arcsin(x)$ , and $\arctan(x)$

**Example**

Calculate  $\int \ln(x) dx$ .

**Solution**

Set  $u = \ln(x)$ ,  $du = \frac{1}{x} dx$  and  $dv = 1 dx$ ,  $v = x$ . Then

$$\int \ln(x) dx = x \ln(x) - \int x (1/x) dx = x \ln(x) - x + C.$$

Verification:

```
[ > testeq(diff(x*ln(x) - x + C, x) = ln(x));
      true
```

**Example**

Calculate  $\int \arcsin(x) dx$ .

**Solution**

Set  $u = \arcsin(x)$ ,  $du = \frac{1}{\sqrt{1-x^2}} dx$  and  $dv = 1 dx$ ,  $v = x$ . Then

$$\int \arcsin(x) dx = x \arcsin(x) - \int \frac{x}{\sqrt{1-x^2}} dx = x \arcsin(x) + (1-x^2)^{\left(\frac{1}{2}\right)} + C.$$

(The final integration is done by substituting  $u = 1 - x^2$ ,  $du = -2x dx$ .)

Verification:

```
[ > testeq(diff(x*arcsin(x) + (1-x^2)^(1/2) + C, x) = arcsin(x));
      true
```

**Example**

Calculate  $\int \arctan(x) dx$ .

### Solution

Set  $u = \arctan(x)$ ,  $du = \frac{1}{1+x^2} dx$  and  $dv = 1 dx$ ,  $v = x$ . Then

$$\int \arctan(x) dx = x \arctan(x) - \int \frac{x}{1+x^2} dx = x \arctan(x) - \frac{1}{2} \ln(1+x^2) + C .$$

(The final integration is done by substituting  $u = 1 + x^2$ ,  $du = 2x dx$ .)

Verification:

```
[ > testeq(diff(x*arctan(x)-1/2*ln(1+x^2) + C, x) = arctan(x));  
      true
```

## 5. Other Integrals Involving $\ln(x)$

### Example

Calculate  $\int x^p \ln(x) dx$ .

### Solution

Set  $u = \ln(x)$ ,  $du = \frac{1}{x} dx$  and  $dv = x^p dx$ ,  $v = \frac{1}{p+1} x^{(p+1)}$ . Then

$$\int x^p \ln(x) dx = \frac{1}{p+1} x^{(p+1)} \ln(x) - \frac{1}{p+1} \int \frac{x^{(p+1)}}{x} dx \text{ or}$$
$$\int x^p \ln(x) dx = \frac{1}{p+1} x^{(p+1)} \ln(x) - \frac{1}{(p+1)^2} x^{(p+1)} + C$$

Verification:

```
[ > testeq( diff(x^(p+1)*ln(x)/(p+1)-x^(p+1)/(p+1)^2, x) =  
      x^p*ln(x) );
```

**Example**

Find a reduction formula for  $\int \ln(x)^p dx$ .

**Solution**

Set  $u = \ln(x)^p$ ,  $du = \frac{p \ln(x)^{(p-1)}}{x} dx$  and  $dv = 1 dx$ ,  $v = x$ . Then

$$\int \ln(x)^p dx = x \ln(x)^p - p \int \ln(x)^{(p-1)} dx.$$

**Example**

Calculate  $\int \ln(x)^3 dx$ .

**Solution**

Using the reduction formula of the preceding example with  $p = 3$ , we have

$$\int \ln(x)^3 dx = x \ln(x)^3 - 3 \int \ln(x)^2 dx.$$

Using the reduction formula of the preceding example with  $p = 2$ , we have

$$\int \ln(x)^2 dx = x \ln(x)^2 - 2 \int \ln(x) dx.$$

Thus,

$$\int \ln(x)^3 dx = x \ln(x)^3 - 3 (x \ln(x)^2 - 2 (x \ln(x) - x)) + C.$$

or

$$\int \ln(x)^3 dx = x \ln(x)^3 - 3 x \ln(x)^2 + 6 x \ln(x) - 6 x + C.$$

Verification:

```
> testeq(diff(x*ln(x)^3-3*x*ln(x)^2+6*x*ln(x)-6*x+C, x) =
ln(x)^3);
```

*true*

### Example

Calculate  $\int \ln((ax+b)^p) dx$ .

### Solution

First we consider  $\int \ln(ax+b) dx$ .

Set  $u = \ln(ax+b)$ ,  $du = \frac{a}{ax+b} dx$  and  $dv = 1 dx$ ,  $v = x$ . Then

$$\int \ln(ax+b) dx = x \ln(ax+b) - \int \frac{ax}{ax+b} dx.$$

**General rule: Whenever integrating  $\frac{p(x)}{q(x)}$  where  $p(x)$  and  $q(x)$  are polynomials and  $\text{degree}(q) \leq \text{degree}(p)$ , divide!**

Since  $\frac{ax}{ax+b} = 1 - \frac{b}{ax+b}$ , we have  $\int \frac{ax}{ax+b} dx = x - \frac{b}{a} \ln(|ax+b|) + C$ .

Thus,

$$\int \ln(ax+b) dx = x \ln(ax+b) - x + \frac{b}{a} \ln(|ax+b|) + C.$$

Finally,

$$\int \ln((ax+b)^p) dx = p \int \ln(ax+b) dx = px \ln(ax+b) - px + \frac{pb}{a} \ln(|ax+b|) + C.$$

Verification:

```
> simplify(diff(p*x*ln(a*x+b) - p*x + p*b/a*ln(abs(a*x+b)) + C,
```

```
x) - ln((a*x+b)^p) assuming a>0, b>0, x>0;
```

```
0
```

### Example

Calculate  $\int \ln(x^2 + 7x + 10) dx$ .

### Solution

We factor the argument of the logarithm:

```
> x^2+7*x+10 = factor( x^2+7*x+10 );
```

$$x^2 + 7x + 10 = (x + 5)(x + 2)$$

Thus,

$$\int \ln(x^2 + 7x + 10) dx = \int \ln(x + 5) dx + \int \ln(x + 2) dx.$$

We apply the result of the preceding example to each integral on the right, using  $p = 1$  and  $a = 1$  for both and  $b = 5$  and  $b = 2$

$$\int \ln(x^2 + 7x + 10) dx = x \ln(x + 5) - x + 5 \ln(|x + 5|) + x \ln(x + 2) - x + 2 \ln(|x + 2|) +$$

C, or

$$\int \ln(x^2 + 7x + 10) dx = x \ln(x^2 + 7x + 10) - 2x + 5 \ln(|x + 5|) + 2 \ln(|x + 2|) + C.$$

Verification:

```
> simplify(
```

```
diff(x*ln(x^2+7*x+10)-2*x+5*ln(abs(x+5))+2*ln(abs(x+2)), x) -  
ln(x^2+7*x+10)) assuming x > 0;
```

```
0
```

**Example (Exercise 35, Page 515 Blank-Krantz)**

Evaluate  $\int \ln(1+x^2) dx$ .

### Solution

Set  $u = \ln(1+x^2)$ ,  $du = \frac{2x}{1+x^2} dx$  and  $dv = 1 dx$ ,  $v = x$ . Then

$$\int \ln(1+x^2) dx = x \ln(1+x^2) - \int \frac{2x^2}{1+x^2} dx.$$

According to the general rule announced above, we divide:

```
[ > quotient := 2*x^2/(1+x^2);
      quotient :=  $\frac{2x^2}{1+x^2}$ 
[ > quotient = quo(numer(quotient), denom(quotient), x) +
      rem(numer(quotient), denom(quotient), x)/denom(quotient);
       $\frac{2x^2}{1+x^2} = 2 - \frac{2}{1+x^2}$ 
```

Thus

$$\int \ln(1+x^2) dx = x \ln(1+x^2) - 2x + \int \frac{2}{1+x^2} dx \quad \text{or}$$

$$\int \ln(1+x^2) dx = x \ln(1+x^2) - 2x + 2 \arctan(x).$$

Verification

```
[ > testeq( diff(x*ln(1+x^2)-2*x+2*arctan(x), x) = ln(1+x^2) );
      true
```

### Example

Express  $\int \ln(ax^2 + bx + c) dx$  in terms of an integral of the form  $\int \frac{\alpha x + \beta}{ax^2 + bx + c} dx$ .

(How we handle the latter depends on whether the denominator is an irreducible quadratic or the product of two linear terms.)

### Solution

Set  $u = \ln(ax^2 + bx + c)$ ,  $du = \frac{2ax + b}{ax^2 + bx + c} dx$  and  $dv = 1 dx$ ,  $v = x$ . Then

$$\int \ln(ax^2 + bx + c) dx = x \ln(ax^2 + bx + c) - \int \frac{2ax^2 + bx}{ax^2 + bx + c} dx.$$

According to the general rule announced above, we divide:

```
> quotient := (2*a*x^2+b*x)/(a*x^2+b*x+c);
```

$$\text{quotient} := \frac{2ax^2 + bx}{ax^2 + bx + c}$$

```
> quotient = quo(quotient, denom(quotient), x) +  
rem(quotient, denom(quotient), x)/denom(quotient);
```

$$\frac{2ax^2 + bx}{ax^2 + bx + c} = 2 + \frac{-bx - 2c}{ax^2 + bx + c}$$

Thus

$$\int \ln(ax^2 + bx + c) dx = x \ln(ax^2 + bx + c) - 2x + \int \frac{bx + 2c}{ax^2 + bx + c} dx.$$

Verification

```
> testeq( diff(x*ln(a*x^2+b*x+c)-2*x, x) = ln(a*x^2+b*x+c) -
```

$((b*x+2*c)/(a*x^2+b*x+c))$ );

true

**Note:**

>  $\text{Int}((\alpha x + \beta)/(a x^2 + b x + c), x) =$   
 $\text{int}((\alpha x + \beta)/(a x^2 + b x + c), x);$

$$\int \frac{\alpha x + \beta}{a x^2 + b x + c} dx =$$

$$\frac{1}{2} \frac{\alpha \ln(a x^2 + b x + c)}{a} + \frac{2 \arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right) \beta}{\sqrt{4 a c - b^2}} - \frac{\arctan\left(\frac{2 a x + b}{\sqrt{4 a c - b^2}}\right) \alpha b}{\sqrt{4 a c - b^2} a}$$

## 5. Other Integrals Involving $\arctan(x)$

**Example**

Calculate  $\int x \arctan(x) dx$ .

**Solution**

Set  $u = \arctan(x)$ ,  $du = \frac{1}{1+x^2} dx$  and  $dv = x dx$ ,  $v = \frac{1}{2} x^2$ . Then

$$\int x \arctan(x) dx = \frac{1}{2} x^2 \arctan(x) - \frac{1}{2} \int \frac{x^2}{1+x^2} dx \quad \text{or}$$

**Following the general rule mentioned above, since  $\text{degree}(1+x^2) \leq \text{degree}(x^2)$ , we divide:**

$$\int x \arctan(x) dx = \frac{1}{2} x^2 \arctan(x) - \frac{1}{2} \int \left(1 - \frac{1}{1+x^2}\right) dx + C$$

Thus,

$$\int x \arctan(x) dx = \frac{1}{2} x^2 \arctan(x) - \frac{1}{2} (x - \arctan(x)) + C$$

Verification:

```
[ > testeq( diff(x^2*arctan(x)/2-x/2+arctan(x)/2, x) = x*arctan(x)
);
                                     true
```

### Example

Calculate  $\int x^2 \arctan(x) dx$ .

### Solution

Set  $u = \arctan(x)$ ,  $du = \frac{1}{1+x^2} dx$  and  $dv = x^2 dx$ ,  $v = \frac{1}{3} x^3$ . Then

$$\int x^2 \arctan(x) dx = \frac{1}{3} x^3 \arctan(x) - \frac{1}{3} \int \frac{x^3}{1+x^2} dx \quad \text{or}$$

Following the general rule mentioned above, since  $\text{degree}(1+x^2) \leq \text{degree}(x^3)$ , we divide:

```
[ > x^3/(1+x^2) = quo(x^3, 1+x^2, x) + rem(x^3, 1+x^2, x)/(1+x^2);
                                     x^3
                                     1+x^2 = x - x
                                     1+x^2
```

$$\int x^2 \arctan(x) dx = \frac{1}{3} x^3 \arctan(x) - \frac{1}{3} \int x - \frac{x}{1+x^2} dx + C$$

Thus,

$$\int x^2 \arctan(x) dx = \frac{1}{3} x^3 \arctan(x) - \frac{1}{3} \left( \frac{x^2}{2} - \frac{\ln(1+x^2)}{2} \right) + C$$

or

$$\int x^2 \arctan(x) dx = \frac{1}{3} x^3 \arctan(x) - \frac{1}{6} x^2 + \frac{\ln(1+x^2)}{6} + C$$

Verification:

```
[ > testeq( diff(x^3*arctan(x)/3-x^2/6+ln(1+x^2)/6, x) =
```

$x^2 \arctan(x)$  ;

*true*

## - 6. Integrating Powers of $\sec(x)$

We know that

$$\int \sec(x) dx = \ln|\sec(x) + \tan(x)| + C \quad \text{and}$$

$$\int \sec(x)^2 dx = \tan(x) + C .$$

### Example

Calculate  $\int \sec(x)^3 dx$  .

### Solution

Set  $u = \sec(x)$ ,  $du = \sec(x) \tan(x) dx$  and  $dv = \sec(x)^2 dx$ ,  $v = \tan(x)$  . Then

$$\int \sec(x)^3 dx = \sec(x) \tan(x) - \int \sec(x) \tan(x)^2 dx \quad \text{or}$$

$$\int \sec(x)^3 dx = \sec(x) \tan(x) - \int \sec(x) (\sec(x)^2 - 1) dx \quad \text{or}$$

$$\int \sec(x)^3 dx = \sec(x) \tan(x) - \int \sec(x)^3 dx + \int \sec(x) dx .$$

Therefore,

$$2 \int \sec(x)^3 dx = \sec(x) \tan(x) + \int \sec(x) dx$$

or

$$\int \sec(x)^3 dx = \frac{1}{2} \sec(x) \tan(x) + \frac{1}{2} \ln|\sec(x) + \tan(x)| + C .$$

## 7. Reduction Formula for Powers of $\sin(x)$

### Exercise

Derive the reduction formula

$$\int \sin(x)^N dx = -\frac{1}{N} \sin(x)^{(N-1)} \cos(x) + \frac{N-1}{N} \int \sin(x)^{(N-2)} dx.$$

### Solution

Set  $u = \sin(x)^{(N-1)}$ ,  $du = (N-1) \sin(x)^{(N-2)} \cos(x) dx$  and  $dv = \sin(x) dx$ ,  $v = -\cos(x)$ .

Then

$$\int \sin(x)^N dx = -\sin(x)^{(N-1)} \cos(x) + (N-1) \int \sin(x)^{(N-2)} \cos(x)^2 dx \text{ or}$$

$$\int \sin(x)^N dx = -\sin(x)^{(N-1)} \cos(x) + (N-1) \int \sin(x)^{(N-2)} (1 - \sin(x)^2) dx \text{ or}$$

$$\int \sin(x)^N dx = -\sin(x)^{(N-1)} \cos(x) + (N-1) \int \sin(x)^{(N-2)} dx - (N-1) \int \sin(x)^N dx .$$

Therefore,

$$(1 + N - 1) \int \sin(x)^N dx = -\sin(x)^{(N-1)} \cos(x) + (N-1) \int \sin(x)^{(N-2)} dx$$

or

$$\int \sin(x)^N dx = -\frac{1}{N} \sin(x)^{(N-1)} \cos(x) + \frac{N-1}{N} \int \sin(x)^{(N-2)} dx .$$

## 8. Reduction Formula for Powers of $\cos(x)$

### Exercise

Derive the reduction formula

$$\int \cos(x)^N dx = \frac{1}{N} \sin(x) \cos(x)^{(N-1)} + \frac{N-1}{N} \int \cos(x)^{(N-2)} dx.$$

### Solution

Set  $u = \cos(x)^{(N-1)}$ ,  $du = -(N-1) \cos(x)^{(N-2)} \sin(x) dx$  and  $dv = \cos(x) dx$ ,  $v = \sin(x)$ .  
Then

$$\int \cos(x)^N dx = \sin(x) \cos(x)^{(N-1)} + (N-1) \int \cos(x)^{(N-2)} \sin(x)^2 dx \text{ or}$$

$$\int \cos(x)^N dx = \sin(x) \cos(x)^{(N-1)} + (N-1) \int \cos(x)^{(N-2)} (1 - \cos(x)^2) dx \text{ or}$$

$$\int \cos(x)^N dx = \cos(x)^{(N-1)} \sin(x) + (N-1) \int \cos(x)^{(N-2)} dx - (N-1) \int \cos(x)^N dx .$$

Therefore,

$$(1 + N - 1) \int \cos(x)^N dx = \sin(x) \cos(x)^{(N-1)} + (N-1) \int \cos(x)^{(N-2)} dx$$

or

$$\int \cos(x)^N dx = \frac{1}{N} \sin(x) \cos(x)^{(N-1)} + \frac{N-1}{N} \int \cos(x)^{(N-2)} dx .$$

## 9. Other Examples

### Example (Exercise 39, Page 515)

Calculate  $\int_{-1}^1 \frac{x}{\sqrt{x+3}} dx.$

### Solution 1

```
> J := Int(x/sqrt(x+3), x = -1 .. 1);
```

$$J := \int_{-1}^1 \frac{x}{\sqrt{x+3}} dx$$

> `K := student[intparts](J, x); #This carries out integration by parts with u = x`

$$K := 2\sqrt{4} + 2\sqrt{2} - \int_{-1}^1 2\sqrt{x+3} dx$$

> `value(K);`

$$2\sqrt{4} + \frac{14\sqrt{2}}{3} - \frac{32}{3}$$

## Solution 2

> `J := Int(x/sqrt(x+3), x = -1 .. 1);`

$$J := \int_{-1}^1 \frac{x}{\sqrt{x+3}} dx$$

> `K := student[changevar](y = x+3, J, y);`

$$K := \int_2^4 \frac{-3+y}{\sqrt{y}} dy$$

> `Int(-3*y^(-1/2), y = 2 .. 4) + Int(y^(1/2), y = 2 .. 4) =  
int(-3*y^(-1/2), y = 2 .. 4) + int(y^(1/2), y = 2 .. 4);`

$$\int_2^4 -\frac{3}{\sqrt{y}} dy + \int_2^4 \sqrt{y} dy = -\frac{20}{3} + \frac{14\sqrt{2}}{3}$$

## Example

Suppose that  $a \neq b$ . Evaluate  $\int \cos(ax) \sin(bx) dx$ .

## Solution

> `J := Int(cos(a*x)*sin(b*x), x);`

$$J := \int \cos(ax) \sin(bx) dx$$

> **J = student[intparts](J, cos(a\*x)); #int by parts with u = cos(a\*x)**

$$\int \cos(ax) \sin(bx) dx = -\frac{\cos(ax) \cos(bx)}{b} - \int \frac{\sin(ax) a \cos(bx)}{b} dx$$

> **A := -cos(a\*x)/b\*cos(b\*x);  
K := Int(sin(a\*x)\*cos(b\*x), x);  
eqn := J = A -a/b\*K;**

$$A := -\frac{\cos(ax) \cos(bx)}{b}$$

$$K := \int \sin(ax) \cos(bx) dx$$

$$eqn := \int \cos(ax) \sin(bx) dx = -\frac{\cos(ax) \cos(bx)}{b} - \left( \frac{a}{b} \int \sin(ax) \cos(bx) dx \right)$$

> **eqn2 := K = student[intparts](K, sin(a\*x)); #int by parts with u = sin(a\*x)**

$$eqn2 := \int \sin(ax) \cos(bx) dx = \frac{\sin(ax) \sin(bx)}{b} - \int \frac{\cos(ax) a \sin(bx)}{b} dx$$

> **B := sin(a\*x)\*sin(b\*x)/b;  
eqn3 := K = B -a/b\*J;**

$$B := \frac{\sin(ax) \sin(bx)}{b}$$

$$eqn3 := \int \sin(ax) \cos(bx) dx = \frac{\sin(ax) \sin(bx)}{b} - \left( \frac{a}{b} \int \cos(ax) \sin(bx) dx \right)$$

> **eqn4 := subs(eqn3, eqn);**

$$eqn4 := \int \cos(ax) \sin(bx) dx =$$

$$-\frac{\cos(ax) \cos(bx)}{b} - \frac{a \left( \frac{\sin(ax) \sin(bx)}{b} - \left( \frac{a}{b} \int \cos(ax) \sin(bx) dx \right) \right)}{b}$$

> **eqn5 := J = solve(eqn4, Int(cos(a\*x)\*sin(b\*x), x));**

$$eqn5 := \int \cos(ax) \sin(bx) dx = \frac{\cos(ax) \cos(bx) b + a \sin(ax) \sin(bx)}{-b^2 + a^2}$$

```
[ > testeq(diff(rhs(eq5), x) = cos(a*x)*sin(b*x));  
                                     true
```

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