Math 132
Fall 2006 Exam II

1. Suppose that $f(x) = 2^{\left(-\frac{1}{x}\right)}$. Calculate D(f)(2).

a) $\frac{\sqrt{2}\ln(2)}{8}$ b) $\frac{\sqrt{2}\ln(2)}{4}$ c) $\frac{\sqrt{2}\ln(2)}{2}$ d) $\sqrt{2}\ln(2)$ e) $2\sqrt{2}\ln(2)$

f) $\frac{\sqrt{2}}{\ln(2)\,8}$ g) $\frac{\sqrt{2}}{\ln(2)\,4}$ h) $\frac{\sqrt{2}}{\ln(2)\,2}$ i) $\frac{\sqrt{2}}{\ln(2)}$ j) $\frac{2\sqrt{2}}{\ln(2)}$

Solution: a

```
> A := diff(2^(-1/x), x);
```

$$A := \frac{2^{\left(-\frac{1}{x}\right)}\ln(2)}{x^2}$$

```
> subs(x=2, A);
```

$$\frac{1}{8}\sqrt{2}\ln(2)$$

2. Calculate $\int_1^{10} \log_{10}(x)\,dx$.

a) $10 - \frac{1}{\ln(10)}$ b) $10 + \frac{1}{\ln(10)}$ c) $10 - \ln(10)$ d) $10 + \ln(10)$ e) $10\ln(10) - 1$

f) $10\ln(10) + 1$ g) $10 - \frac{9}{\ln(10)}$ h) $1 + \frac{10}{\ln(10)}$ i) $\log_{10}(10)$ j) $\log_{10}(10) - 1$

Solution: g

```
> int(log[10](x),x=1..10);
```

$$\frac{-9 + 10\ln(2) + 10\ln(5)}{\ln(2) + \ln(5)}$$

```
> testeq(int(log[10](x),x=1..10) = 10-9/ln(10));
```

$$true$$

3. Suppose that $f(x) = x^{\sqrt{x}}$. Calculate D(f)(4).

a) $\sqrt{2}+1$ b) $2\sqrt{2}+1$ c) $4\sqrt{2}+1$ d) $\sqrt{2}+\ln(2)$ e) $2\sqrt{2}+\ln(2)$
f) $4\sqrt{2}+\ln(2)$ g) $\ln(2)+1$ h) $\ln(2)+2$ i) $8(\ln(2)+1)$ j) $\sqrt{2}\ln(2)$

Solution: i

```
> diff(x^sqrt(x), x);
```

$$x^{(\sqrt{x})}\left(\frac{1}{2}\frac{\ln(x)}{\sqrt{x}}+\frac{1}{\sqrt{x}}\right)$$

```
> simplify(subs(x=4, %));
```

$$8\ln(2)+8$$

4. A radioactive substance is measured. Only 1/3 of the substance remains 3 hours later. Express the half-life of this substance in terms of the quantities $\alpha = \ln(2)$ and $\beta = \ln(3)$.

a) $\beta-\alpha$ b) $e^{\alpha}\beta-\alpha$ c) $\frac{\alpha}{\beta-\alpha}$ d) $\frac{\beta}{\beta-\alpha}$ e) $\frac{e^{\alpha}\alpha}{\beta-\alpha}$

f) $\frac{e^{\alpha}\beta}{\alpha}$ g) $\frac{\beta}{\alpha}$ h) $\frac{\alpha}{\beta}$ i) $\frac{e^{\alpha}\alpha}{\beta}$ j) $\frac{e^{\beta}\alpha}{\beta}$

Solution: j

```
> m := t -> m0/2^(t/tau);
```

$$m := t \rightarrow \frac{m0}{2^{\left(\frac{t}{\tau}\right)}}$$

```
> eqn := m(3) = m(0)/3;
```

$$eqn := \frac{m0}{2^{\left(\frac{3}{\tau}\right)}} = \frac{m0}{3}$$

```
> tau = solve(eqn, tau);
```

$$\tau = \frac{3 \ln(2)}{\ln(3)}$$

5. The mass of an Eek. oli colony is growing exponentially and is increasing at the rate of 20 µg/hr when the colony's mass is 60 µg. How many hours is the half-life?

a) ln(2) b) 2 ln(2) c) 3 ln(2) d) 6 ln(2) e) 12 ln(2)
f) 1/ln(2) g) 2/ln(2) h) 3/ln(2) i) 6/ln(2) j) 12/ln(2)

Solution: c

```
> m := t -> m0*2^(t/tau);
```

$$m := t \rightarrow m0\, 2^{\left(\frac{t}{\tau}\right)}$$

```
> D(m)(t);
```

$$\frac{m0\, 2^{\left(\frac{t}{\tau}\right)} \ln(2)}{\tau}$$

```
> eqn := 'D(m)(t)' = 'm(t)'*ln(2)/tau;
  #Notice that m(t) = m0*2^(t/tau) is a factor of D(m)(t)
```

$$eqn := \mathrm{D}(m)(t) = \frac{\mathrm{m}(t) \ln(2)}{\tau}$$

```
> eqn2 := 20*mu*g/hr = 60*mu*g*ln(2)/tau;
```

$$eqn2 := \frac{20\, \mu\, g}{hr} = \frac{60\, \mu\, g \ln(2)}{\tau}$$

```
> answer := tau = solve(eqn2, tau);
```

$$answer := \tau = 3 \ln(2)\, hr$$

6. An intravenous drip delivers a drug to a patient so that the drug is absorbed into the patient's bloodstream at the constant rate of 60 mg/hr. Drip, drip, drip, drip, drip, and so on. The drug is eliminated from the bloodstream
at a rate proportional to the mass of the drug in the bloodstream. If the mass is measured in mg, then the proportionality constant is 5/hr. (Once you write out the differential equation that the mass satisfies, you will see that the proportionality constant bears the units of 1/time. Its numerical value depends on the units chosen.) Long term, what, approximately, is the nearly constant mass of the drug maintained in the patient's

bloodstream?

a) 1 b) 2 c) 3 d) 4 e) 5

f) 6 g) 10 h) 12 i) 15 j) 20

Solution: h

```
> dsolve({diff(u(t),t) = 60-5*u(t), u(0) = u0},u(t));
```

$$u(t) = 12 + e^{(-5\,t)}\,(-12 + u0)$$

```
> limit(rhs(%), t = infinity);
```

$$12$$

7. Suppose that $f(x) = \arcsin(\sqrt{x})$. Calculate $D(f)\left(\frac{1}{2}\right)$. (The derivative of f(x) at $x = \frac{1}{2}$).

a) 1 b) $\sqrt{2}$ c) $\frac{1}{\sqrt{2}}$ d) $\sqrt{3}$ e) $\frac{2}{\sqrt{3}}$

f) $\frac{1}{2}$ g) 2 h) $2\sqrt{2}$ i) $2\sqrt{3}$ j) 4

Solution: a

```
> D(x -> arcsin(sqrt(x)) )(1/2);
```

$$1$$

8. Suppose that $f(x) = \arctan(x)$. What is $D(f)\left(\frac{1}{\sqrt{3}}\right)$? (The derivative of f(x) at $x = \frac{1}{\sqrt{3}}$).

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{3}$ d) $\frac{1}{4}$ e) $\frac{1}{\sqrt{3}}$

f) $\frac{\sqrt{3}}{2}$ g) $\sqrt{3}$ h) $2\sqrt{3}$ i) $\frac{3}{4}$ j) $\frac{4}{3}$

Solution: i

```
> D(arctan)(1/sqrt(3));
```

$$\frac{3}{4}$$

9. Calculate $\int_0^1 x\, e^x\, dx$.

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{3}{4}$ d) 1 e) 2

f) e g) $e-1$ h) $e-2$ i) $2e-1$ j) $2e+1$

Solution: d

```
> int(x*exp(x),x=0..1);
```

$$1$$

10. Calculate $\int_0^{\pi} x^2 \sin(x)\, dx$.

a) $\pi^2+\frac{1}{2}$ b) π^2+1 c) π^2+2 d) π^2+4 e) π^2+5

f) $\pi^2-\frac{1}{2}$ g) π^2-1 h) π^2-2 i) π^2-4 j) π^2-5

Solution: i

```
> int(x^2*sin(x),x = 0 .. Pi);
```

$$\pi^2-4$$

11. Calculate $36\int_1^{e} x^5 \ln(x)\, dx$.

a) e^6-1 b) $2e^6-1$ c) $3e^6-1$ d) $4e^6-1$ e) $5e^6-1$

f) e^6+1 g) $2e^6+1$ h) $3e^6+1$ i) $4e^6+1$ j) $5e^6+1$

Solution: j

```
> 36*int(x^5*ln(x),x = 1 .. exp(1));
```

$$5\,e^{6}+1$$

12. Calculate $\displaystyle\int_{2}^{3}\frac{x+3}{x^{2}-1}\,dx$.

a) $\ln(2)$ b) $\ln(3)$ c) $2\ln(2)$ d) $2\ln(3)$ e) $3\ln(2)$

f) $\ln(6)$ g) $\ln\left(\frac{2}{3}\right)$ h) $\ln\left(\frac{3}{2}\right)$ i) $\ln\left(\frac{9}{2}\right)$ j) $\ln(12)$

Solution: b

```
> int((x+3)/(x^2-1),x = 2 .. 3);
```

$$\ln(3)$$

13. Calculate $\displaystyle\int_{2}^{3}\frac{5x^{2}-x-1}{x^{2}(x-1)}\,dx$.

a) $\ln(12)-\frac{1}{3}$ b) $\ln(6)-\frac{1}{2}$ c) $\ln(18)+\frac{1}{6}$ d) $2\ln(3)+\frac{1}{4}$ e) $3\ln(2)-\frac{1}{2}$

f) $\ln(6)-\frac{2}{3}$ g) $2\ln(4)-\frac{1}{4}$ h) $\ln(24)+\frac{1}{3}$ i) $\ln(12)-\frac{1}{6}$ j) $\ln(8)+\frac{1}{3}$

Solution: c

```
> int((5*x^2-x-1)/(x^2)/(x-1),x = 2 .. 3);
```

$$\frac{1}{6}+\ln(2)+2\ln(3)$$

```
> testeq(A = ln(9/2)-1/6);
```

$$true$$

14. Calculate $\int_0^{\frac{\pi}{2}} \sin(x)^3 \, dx$.

a) 1/15 b) 2/15 c) 1/5 d) 4/15 e) 1/3
f) 2/5 g) 7/15 h) 8/15 i) 3/5 j) 2/3

Solution: j

```
> int(sin(x)^3,x = 0 .. Pi/2);
```

$$\frac{2}{3}$$

15. From the reduction formula

$$\int \cos(x)^n \, dx = \frac{\sin(x)\cos(x)^{(n-1)}}{n} + \frac{(n-1)\int \cos(x)^{(n-2)} \, dx}{n}$$

it follows that there are rational numbers A, B, and C such that

$$\int \cos(x)^4 \, dx = A \cos(x)^3 \sin(x) + B \cos(x) \sin(x) + C\, x + \mathrm{D}$$

where D is a constant. What is C?

a) 8/15 b) 2/3 c) 7/9 d) 3/8 e) 5/7
f) 24/35 g) 16/21 h) 9/16 i) 3/4 j) 3/5

Solution: d

```
> eqn1 := int(cos(x)^4,x);
```

$$eqn1 := \frac{1}{4}\cos(x)^3\sin(x) + \frac{3}{8}\cos(x)\sin(x) + \frac{3x}{8}$$

16. Calculate $\displaystyle\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\,dx.$

a) 1/3	b) 2/3	c) 3/4	d) 3/5	e) 4/5
f) 1/4	g) 1/2	h) 1/8	i) 3/8	j) 5/8

Solution: b

```
> int(x^3/sqrt(1-x^2),x=0..1);
```

$$\frac{2}{3}$$

17. Calculate $\displaystyle\int_0^1 \frac{1}{(1+x^2)^{\left(\frac{3}{2}\right)}}\,dx.$

a) $\frac{1}{2}\ln(2)$	b) $\ln(2)$	c) $2\ln(2)$	d) $\frac{1}{2}\ln(3)$	e) $\ln(3)$
f) $2\ln(3)$	g) $3\ln(3)$	h) 1	i) 2	j) $\frac{1}{\sqrt{2}}$

Solution: j

```
> int(1/(1+x^2)^(3/2),x = 0 .. 1);
```

$$\frac{\sqrt{2}}{2}$$

18. There are unique rational numbers A and B such that

$$\int_{\frac{1}{\sqrt{3}}}^{1} \frac{4x^2 - x + 4}{x(x^2+1)}\,dx = A\ln(3) + B\pi.$$

What is A ?

a) –5 b) –4 c) –3 d) –2 e) –1
f) 5 g) 4 h) 3 i) 2 j) 1

Solution: i

```
> int(-1/(x^2+1)+4/x, x = 1/sqrt(3)..1);
```

$$-\frac{\pi}{12}+2\ln(3)$$

```
> int((4*x^2-x+4)/x/(x^2+1),x=1/sqrt(3)..1);
```

$$-\frac{\pi}{12}+2\ln(3)$$

19. The region in the first quadrant bounded above by $y=2-x$ and below by $y=\sqrt{x}$ is rotated about the x-axis. What is the volume of the resulting solid of revolution?

a) $\frac{12\pi}{5}$ b) $\frac{31\pi}{15}$ c) $\frac{15\pi}{8}$ d) $\frac{8\pi}{3}$ e) $\frac{16\pi}{5}$
f) $\frac{11\pi}{6}$ g) $\frac{5\pi}{3}$ h) $\frac{9\pi}{5}$ i) $\frac{32\pi}{15}$ j) $\frac{21\pi}{8}$

Solution: f

```
> Pi*int((2-x)^2-(sqrt(x))^2,x=0..1);
```

$$\frac{11\pi}{6}$$

20. The region above the x-axis and under the graph of $y=\sin(\pi x^2)$, $0 \le x \le 1$ is rotated about the y-axis. What is the volume of the resulting solid of revolution?

a) 1 b) 2 c) 3 d) 4 e) 5
f) π g) $\frac{3\pi}{2}$ h) 2π i) 3π j) 4π

Solution: b

```
> int(2*Pi*x*sin(Pi*x^2),x=0..1);
```

$$2$$