M233 Spring 2004 Homework Assignment 2

Due: 23 February 2004

1. Sketch the graph of \( r(t) = t^2i - \ln(t)j + 2tk \) for \( \frac{1}{100} \leq t \leq 2 \). Calculate the velocity vector \( r'(1) \), the unit tangent vector \( T(1) \), and the tangent line to the curve at the point \( P_0 = (1, 0, 2) \). At what point does this tangent line intersect the \( xz \)-plane?

2. What Cartesian equation describes the normal plane to the curve \( r(t) = t^2i - \ln(t)j + 2tk \) at \( P_0 = (1, 0, 2) \)?

3. Calculate the arc length of \( r(t) = t^2i - \ln(t)j + 2tk \) between the points \( (1, 0, 2) \) and \( (e^2, -1, 2e) \).

4. Calculate the principal unit normal vector \( N \) to \( r(t) = t^2i - \ln(t)j + 2tk \) at \( P_0 = (1, 0, 2) \). State symmetric equations for the normal line to \( r(t) \) at \( P_0 = (1, 0, 2) \). (This is the line through \( P_0 \) with direction vector \( N \).) Calculate the unit binormal vector \( B \) to \( r(t) = t^2i - \ln(t)j + 2tk \) at \( P_0 = (1, 0, 2) \). Find a Cartesian equation for the osculating plane of \( r \) at \( P_0 = (1, 0, 2) \).

5. Calculate the curvature of \( r(t) = t^2i - \ln(t)j + 2tk \) at \( P_0 = (1, 0, 2) \). Describe the circle of curvature of \( r \) at \( P_0 = (1, 0, 2) \).