M233 Spring 2004 Homework 3

Due: 19 March 2004

1. For the function $f(x,y) = x \ln(xy)$ and point $(x_0, y_0) = (1, e)$, calculate $\varphi(x) = f(x, y_0)$, $\psi(y) = f(x_0, y)$, $\varphi'(x_0)$, and $\psi'(y_0)$. Then calculate $(\partial f/\partial x)(x_0, y_0)$ and $(\partial f/\partial y)(x_0, y_0)$. Verify that $\varphi'(x_0) = (\partial f/\partial x)(x_0, y_0)$ and $\psi'(y_0) = (\partial f/\partial y)(x_0, y_0)$.

2. For the function $f(x,y) = x \ln(xy)$ and point $(x_0, y_0) = (1, e)$, verify that $f_{xy}(x_0, y_0) = f_{yx}(x_0, y_0)$.

3. The plane $y = e$ intersects the graph of $z = x \ln(xy)$ in a curve. Parameterize the tangent line to the curve at the point $(1, e, 1)$. The plane $x = 1$ intersects the graph of $z = x \ln(xy)$ in another curve. Parameterize the tangent line to this curve at the point $(1, e, 1)$.

4. Find an equation for the tangent plane to the graph of $z = x \ln(xy)$ at $(1, e, 1)$.

5. Suppose that
$$x = s^3 e^{st},$$
$$y = st^2 + e^{1+t}$$
and
$$z = x \ln(xy).$$
These three equations determine a function $z(s,t)$. Use the Chain Rule to calculate
$$\frac{\partial z}{\partial s}(1,0) \quad \text{and} \quad \frac{\partial z}{\partial t}(1,0).$$