Measuring Autonomy Freedom*

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Abstract

In the measurement of autonomy freedom, the admissible potential preference relations are elicited by means of the concept of 'reasonableness'. In this paper we argue for an alternative criterion based on information about the decision maker's 'awareness' of his available opportunities. We argue that such an interpretation of autonomy fares better than that based on reasonableness. We then introduce some axioms that capture this intuition and study their logical implications. In the process, a new measure of autonomy freedom is characterized, which generalizes some of the measures so far constructed in the literature.

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1 Introduction

A recent literature in normative economics, the Freedom of Choice Literature (FCL), has put forward a number of alternative criteria for assessing the extent of opportunity that an individual enjoys. Apart from few exceptions (Sugden 1998, Bavetta and Guala 2003), authors have paid scant attention to the reasons why we should be interested in having opportunity. They take for granted that having opportunity is important and that measuring it, consequently, is a relevant exercise; but why this is so and how different reasons for attributing importance to opportunity impinge on its measurement seems to be an uncommon perspective in FCL.

In this paper we distinguish two reasons for attributing value to having opportunity. The first is rooted in the view that access to alternatives to choose from is a dimension of the positive freedom of an individual. Opportunity is then valued for its own sake, because choosing an option from a set that contains other alternatives reflects an individual's *opportunity freedom*. We shall call such a reason for valuing opportunity *substantive*. Yet, having opportunity may also be valued for *instrumental (procedural)* reasons. This is the case in the Millian tradition (Mill 1859) where having the possibility of selecting among different alternatives is important for its consequences on the development and exercise of one's own individuality. If access to opportunity is important for instrumental reasons, its measure shall be called *autonomy freedom*.

This paper pertains to the instrumental value tradition: our concern is, in particular, with the measurement of the procedural value of opportunity. Existing measures (Pattanaik and Xu 1998, Romero Medina 2001, Sugden 1998) suggest that a quantitative assessment of autonomy freedom requires information about potential preferences. Though we agree with such a view, we depart from the existing results in two respects: first, we reject their sociologically–based interpretation of potential preferences; second, we give a far greater role to information about actual opportunities. We then propose an alternative measure that does not suffer from the same difficulties as it relies on an entirely subjective criterion—'awareness of the circumstances of the choice'—and on the available options. The interpretive foundations of our proposal have been laid out in Bavetta and Guala (2003); in this paper our main concern is with the necessary and sufficient conditions for the existence of a measure.

The paper is organized as follows. Section 2 briefly reviews the two reasons for attributing value to having opportunity. Section 3 unfolds the intuition behind the paper. It introduces the theoretical framework in which measures of autonomy freedom can be constructed; presents and criticizes the solution given so far in the literature; illustrates and defends our proposal for an alternative perspective. Section 4 sets the ground for the construction of our ranking by introducing the axiomatic formulation for the measurement of opportunity that we propose. The last point in our analysis, accomplished in section 5, puts our result in relation with others proposed in the literature. Finally, section 6 recapitulates and suggests lines for further research.

2 The value of opportunity

An analytically useful distinction can be drawn between a *procedural* and a *substantive* value of having opportunity. The latter assigns importance to availability of opportunity irrespective of the choice process. Let X be a finite universe of opportunities and $\mathcal{P}(X) = 2^X \setminus \{\emptyset\}$ its power set whose elements we denote as $A, B, \ldots$. Let also '$\succeq$' be a binary relation on $\mathcal{P}(X)$ and read $A \succeq B$ as 'A gives at least as much opportunity freedom as B'.

In an important contribution to the measurement of opportunity Pattanaik and Xu (1990) introduce the following monotonicity axiom (MON). Given an opportunity set A in $\mathcal{P}(X)$ and an opportunity x which does not belongs to A, we have that $A \cup \{x\} \succeq A$. MON says that access to any further opportunity x enlarges the decision maker's opportunity freedom irrespective of whether the newly acquired alternative would ever be chosen or, at least, considered for selection by the decision maker in the deliberative process. Pattanaik and Xu's monotonicity axiom is a formulation of the substantive value of having opportunity since any newly acquired alternative contributes to the decision maker's opportunity freedom as it expands her menu of alternatives. No reference whatsoever is made to the deliberative process in which the agent would be engaged by the acquisition of the new alternative since the axiom makes no mention to her preferences over the available alternatives. It is then fair to say that, in their framework, the evaluation of the extent of opportunity is *a–deliberative*.

Pattanaik and Xu's is not the only possible formulation of the substantive value of having opportunity. Consider Sen's preference–based monotonicity axiom which says that, for all $x, y \in X$, if x is weakly preferred to y, then $\{x, y\} \succeq \{y\}$ (Sen 1988, 1991, 1993). In this case the new alternative's impact on the enlargement of the agent's extent of opportunity depends both on being distinguishable from the other opportunities and on being preferable. Nonetheless, the value of having opportunity is still entirely substantive since newly acquired alternatives might affect the agent's opportunity freedom but make no difference to her deliberative process. The reason is simple: although Sen's preference–based monotonicity avails itself of information about the decision maker's preferences, such information takes the agent's preference relation as exogenously given, leaving no space for the analysis of the deliberation process, i.e., the process of formation of one's own preferences over a menu of alternatives. It is fair to say then, that Sen's analysis takes place at the *post–deliberation* stage of a choice and his measure is still a measure of opportunity freedom.

Availability of opportunities may also be valuable for procedural reasons. If Susan prefers her work in the bank but the local restaurant would commit her to less hours, having opportunity becomes valuable since it would confront her with alternatives whose selection requires a careful weighting of their respective merits. The process of forming one's own preferences and selecting among alternatives is important since it contributes to the development of Susan's moral and personal qualities and makes her own life the product of her choices. From the procedural point of view then, having opportunity becomes valuable for its impact on the deliberative process rather than for its opportunity freedom enhancing properties.

A major figure in the analysis of the procedural value of having opportunity, John Stuart Mill, in his *On Liberty*, writes that having alternatives to choose from fosters the development of individuality since certain fundamental qualities of an agent which render him autonomous, such as "perception, judgement, discriminative feeling, mental activity, and even moral preference" (Mill 1859, p.122), can only be exercised and developed when he makes choices, i.e., in the deliberation process. More recently the procedural value of having opportunity has been the subject of renovated attention, in particular by Nozick (1974) and Raz (1986), though with a different emphasis with respect to Mill. Rather than restricting it to the 'exercise aspect' of autonomy, Nozick and Raz highlight the fact that the deliberative process may be relevant as it allows the decision maker to shape her life according to her own plan of giving meaning to it.

Note that, in both the Millian and the Nozickean perspective, the value of having opportunity does not rest on the mere fact of having an alternative available but is the consequence of the decision maker's engagement in a deliberative process since choosing is *instrument for* (Mill) and *expression of* (Nozick) a person's autonomy. Choices that induce a decision maker to resort to those personal traits of character, that contribute to the formation and the expression of her own individuality shall be called *relevant* in this paper.

3 Theoretical basis of a measure of autonomy freedom

3.1 Deliberation and potential preferences

It is fair to say that most of the freedom of choice literature, but for fewer than a handful exceptions (Pattanaik and Xu 1998, Sugden 1998, Romero Medina 2001, Bavetta and Guala 2003), has focused on the substantive reasons for attributing value to opportunity and therefore on measures of opportunity freedom. In this paper we wish to explore the case in which having opportunity is valuable for procedural reasons and to construct a measure of autonomy freedom that presents some appealing features.

Whereas the substantive value of having opportunity can be captured by measures that are either a–deliberative or conducted at the post–deliberation stage of a choice, the construction of a measure of opportunity with procedural value can only take place at the *pre–deliberation* stage. The reason is that, since at this stage the deliberative process is still to be accomplished, the decision maker has a genuine possibility for shaping autonomously her own preferences, her own view over the available opportunities; a possibility that is denied at the post–deliberation stage of a choice (such as Sen's) and alien to Pattanaik and Xu's a–deliberative setting.

But, how can the process of developing and affirming one's own autonomy be captured? The answer is by gathering information on the menu of available alternatives *and* on the set of potential preference relations that a decision maker confronts. Recall Mill: shaping and affirming one's own autonomy requires reference to those personal and moral qualities of a person that render her autonomous. Qualities that can only emerge if the decision maker may have 'good reasons' for choosing an alternative instead of another. Potential preference are instrumental to have a grip on 'good reasons'. If two opportunities may be chosen on the basis of two different preference relations, then the decision maker has to refer to her fundamental qualities for deliberating and therefore the choice they call for is a relevant one.

As Sugden (1998) suggests, the difficulty with potential preferences is that they need to be interpreted. They require, to say it differently, some criterion (or screening device) for distinguishing potential from conceivable preferences, if we wish to avoid the Scylla and Charybdis of degenerate or trivial measures of opportunity with procedural value. Confronted with this difficulty, Sugden (1998) proposes a solution that we find unsatisfactory. He suggests that, in constructing the screening device, "[t]here seems to be no way of avoiding appeal to contestable ideas of 'normal', 'reasonable' or 'natural' preferences" (p. 324).

In Sugden's view, reasonableness refers to a 'sociological' interpretation of potential preference. Take a group of individuals identified on the basis of some characteristics other than what they prefer (say, the middle-aged British men, to use his example). Though each individual possesses his own preference relations, they may differ from person to person. Nonetheless, it is likely that, as these relations are expressions of individuals sharing some common pool of characteristics or values, then every person will consider each of them as a preference relation that he may hold. The set of potential preferences is therefore composed by the preference relations of all individuals of a given reference class of people.

3.2 Criticizing reasonableness

The question that lies at the heart of this paper is whether reasonableness is the right candidate as a screening device. Following Bavetta and Guala (2003), we argue that it is not for the reasons that will be illustrated in this section. Consider, to start with, an example.

Example 3.1 *In a given town the local two–screen cinema features quite a di-*

verse program. In screen one patrons can enjoy the latest release of Schwarzenegger's fight for rescuing the globe and its special effects (S). On the contrary, screen two proposes one of the many Truffaut's cerebral analysis of women (T). Two persons, i and j, approach the counter for buying a ticket. Their opportunity set is the same: $\{S, T\}$. Yet, individual i is only aware of Hollywood productions and, therefore, he cannot express a preference for Truffaut, despite Catherine Deneuve playing the leading role. On the contrary, individual j is a regular film-goer fully aware of both the refined direction and acting that characterize Truffaut's movies, as well as the breath-taking plots of Schwarzenegger's adventures.

Suppose we use reasonableness as a screening device. Then, if i and j belong to the same reference class of individuals, a preference for T has to be in i's set of potential preferences. Accordingly, a relevant choice is available to both people despite the contrary evidence that i is in fact unaware of one kind of movies and therefore can hardly face a relevant choice. The use of reasonableness is here at odds with intuition about the extent of autonomy freedom comparisons. On the contrary, if i and j do not belong to the same reference class of individuals, then the reasonableness–based framework leaves the question of their relative degrees of autonomy freedom unanswered as the two situations are incomparable despite the fact that i enjoys the possibility of making a truly relevant choice whereas j does not.

What the example suggests is that if we want to asses how much autonomy freedom an agent enjoys, our measure should embody information about the preferences that the decision maker is *aware of* given his own individual circumstances at the moment of making his choice. These circumstances—not any sociologically determined criterion such as reasonableness—should be the guideline for eliciting potential preference relations and, ultimately, relevant choices.

3.3 Defending awareness

The adoption of *awareness* as the screening device for the measurement of autonomy freedom requires further clarification. First of all, according to some philosophers, to be autonomous an agent must be able to reflect critically upon her preferences, evaluate them and act on the basis of such an evaluation. For example, as argued by Dworkin (1988), autonomous agents are able to engage in a three–step process: 1) identification of the action; 2) appraisal of the action and its motivations; 3) revision of her behavior in the light of the appraisal. Autonomy is then linked to a process of *conscious evaluation* of which awareness, in the sense used in this paper, is an aspect. In our view then, an individual enjoys autonomy freedom to the extent to which he or she may accomplish conscious evaluations of the available alternatives and root his or her choice on such an evaluative process.

Consider the movie example once again. Upon reflection the difference between the two individuals, i and j is nothing but what they know. Notice that knowledge is not connected here to ignorance of the available opportunities: both i and j know that there are two movies on show. But, i's ignorance of French cinema does not allow him to engage in a process of conscious evaluation of the two alternatives and, ultimately, deprives him of the possibility of making a truly autonomous choice. Individual i is negatively free to watch Truffaut's film as well as he is empowered with the resources to buy the ticket and therefore fully endowed with opportunity freedom. Yet, in the sense stated in this paper, he is undoubtedly deprived of autonomy because of the different deliberation process that might lead him to choose Truffaut with respect to individual j.

In another of his books John Stuart Mill provides even further support. The suggestion that informed preferences should be at the basis of autonomy is foreshadowed in Mill's claim that the satisfaction of 'lower' preferences should be discounted in making interpersonal comparisons of utility: "And if the fool, or the pig, are of a different opinion that is because they only know their side of the question" (Mill 1863, p. 140). Without proposing an exegetical claim here, Mill's passage confirms that the perspective on autonomy freedom that we are proposing is a normatively valid one and one that should be distinguished from the notion of autonomy supported by reference to reasonableness as a screening device.

Of course we do not claim that our is the 'only' approach to the notion of being autonomous and to its measurement. In some sense, were i choosing T, he could make a choice that is expression of his autonomy, as all choices are that constitute a 'leap into the Great Unknown'. At a first sight, this consideration might deprive our framework of most of its grip. Yet, there are good reasons for excluding unknown choices from our metric of autonomy freedom. For example, it may be important to distinguish between 'choosing x' and 'having a preference for choosing x'. Were j deciding that the time is ripe for leaping into the unknown, his autonomy would consist of the 'exercise of autonomous judgement in *deciding to choose* T' instead of the 'exercise of autonomous judgement in *choosing* T'. If so, the notions of autonomy implied by the two choices differ and the counterexample misses the point.

4 The analytical framework

To characterize formally and to assess our understanding of autonomy freedom we need information about the available opportunities and the available potential preferences. Such an informational requirement goes beyond that used in other rankings of autonomy freedom such as, for example, Pattanaik and Xu (1998), which is tied to the sociological interpretation of the screening device. We therefore define an alternative space, that we call the space of *opportunity* *situations*. An opportunity situation is a pair composed of information about the set of alternatives available to a decision maker and about the set of all available potential preference relations that he is confronting in a specific choice situation (contextual circumstance).

As we now show, opportunity situations are amenable to formal representation. In particular, on their basis we construct a ranking with some attractive properties that support our view of autonomy freedom. As it turns out, this ranking also generalizes some of the results achieved so far in the literature.

4.1 Notation

To capture our idea of autonomy, we make use of information beyond the mere availability of opportunities. In particular, let $N = \{1, \ldots, n\}$ be the set of agents; we introduce the decision maker's preferences (R), represented by complete and transitive binary relations defined over X. The set of all possible preference relations is denoted by $\Pi = \{R_1, \ldots, R_m\}$. Thus, $\forall x, y \in X$, $\forall h \in \{1, ..., m\}$, xR_hy means that "x is at least as good as y according to the preference ordering R_h". In our framework each of the n individuals in the society holds a set $\Pi_i, i \in N$, of preference orderings, where $\Pi_i \subseteq \Pi$. $\mathcal{P}(\Pi) = 2^{\Pi} \setminus \{\emptyset\}$ represents the set of all subsets of preference relations. Finally, we denote by $|A|$ the cardinality of set A.

We are interested in ranking *opportunity situations*. Formally, an opportunity situation is a pair $(A, \Pi_i) \in \mathcal{P}(X) \times \mathcal{P}(\Pi)$. Hence our ranking is represented by a binary relation '$\succeq$' on $\mathcal{P}(X) \times \mathcal{P}(\Pi)$. The expression $(A, \Pi_i) \succeq (B, \Pi_j)$ should then be read as "the opportunity situation (A, Π_i) offers at least as much autonomy freedom as the opportunity situation (B, Π_j)".

Given a set of opportunities and a set of preference relations, the *choice set* of an agent is defined as the set of preference–wise undominated opportunities on the basis of at least a potential preference relation that the decision maker may hold. So, $\forall A \in \mathcal{P}(X), \forall \Pi_i \in \mathcal{P}(\Pi)$, we denote this set by $max_i(A) \equiv \{x \in A : \exists R \in \Pi_i \text{ such that } xRy \ \forall y \in A\}$.

Next we impose some axioms on $\succeq$, which capture our intuition on the extent of autonomy freedom enjoyed by the decision maker under alternative situations. We assume that the binary relation $\succeq$ is transitive.

4.2 Axioms and the ranking rule

The first axiom establishes the circumstances under which an individual enjoys no autonomy freedom.

Axiom 4.1 *Indifference between no-freedom situations (INF)*
$\forall (A, \Pi_i), (B, \Pi_j) \in \mathcal{P}(X) \times \mathcal{P}(\Pi), \forall x, y \in X$, $[max_i(A) = \{x\}$ *&* $max_j(B) = \{y\}] \Rightarrow (A, \Pi_i) \sim (B, \Pi_j)$.

Axiom INF is related in spirit to the *principle of no choice* introduced by Jones and Sugden (1982) and subsequently used by Pattanaik and Xu (1990), which can be stated as follows: if two opportunity sets are singleton, then the degree of opportunity freedom they offer is identical. The intuition advanced for the property was that, since singleton opportunity sets do not offer any choice at all, then they must be indifferent in terms of freedom, even if the alternatives that they contain could be ranked differently in terms of utility. We believe that this is a very reasonable intuition; however, the principle of no choice, as formulated by Jones and Sugden (1982), fails to detect situations in which lack of autonomy freedom is due to unawareness, rather than to the absence of opportunities to choose from.

This consideration leads us to rephrase this property in such a way as make use of information about opportunity sets and the possibility of making conscious evaluations or informed choices. The idea can be illustrated by the following example.

Example 4.1 *Let* $A, B, C \in \mathcal{P}(X)$ *and* $A = \{x, y, z\}$, $B = \{s\}$ *and* $C = \{t\}$. *Let* k *be an injection from* $\mathcal{P}(X)$ *to* N *and let* $\Pi_i, \Pi_j, \Pi_h \in \mathcal{P}(\Pi)$ *be each agent's set of preference relations. Assume that* $|\Pi_i| = 1$. *If its element is linear, then* $|max_i(A)| = |max_j(B)| = |max_h(C)| = 1$. *The three opportunity situations* (A, Π_i), (B, Π_j) *and* (C, Π_h) *are freedom-wise indistinguishable (they do not offer any choice at all).*

Note the difference between our result and Jones and Sugden's: whereas we require indifference between (A, Π_i) and (B, Π_j) and between (A, Π_i) and (C, Π_h), their principle of no–choice would hold just (B, Π_i) and (C, Π_h) as indifferent. Hence we introduce axiom INF, which states that, if two opportunity situations lead to as many choice sets which are singletons, then the extent of autonomy freedom they offer is the same (being nil). The cardinality of the opportunity sets is therefore irrelevant whenever the preference relations lead to choice sets that are singleton.

The next two properties impose some restrictions on the effect of adding (subtracting) an alternative to (from) a given opportunity set, over the ranking of opportunity situations.

Axiom 4.2 *Addition of undominated alternatives (AUA)*
$\forall (A, \Pi_i) \in \mathcal{P}(X) \times \mathcal{P}(\Pi), \forall x \in X \backslash A, [\exists R \in \Pi_i \textit{ s.t. } \forall y \in A \; xRy] \Rightarrow (A \cup \{x\}, \Pi_i) \succ (A, \Pi_i)$

Axiom 4.3 *Addition of dominated alternatives (ADA)*
$\forall (A, \Pi_i) \in \mathcal{P}(X) \times \mathcal{P}(\Pi), \forall x \in X \backslash A, [\neg \exists R \in \Pi_i \textit{ s.t. } \forall y \in A \; xRy] \Rightarrow (A \cup \{x\}, \Pi_i) \sim (A, \Pi_i)$.

Axioms AUA and ADA describe what happens when an alternative is added to a given opportunity set. Does the addition of a new alternative enhance the degree of freedom enjoyed by an individual? Different answers have been provided in the literature to this question. Pattanaik and Xu (1990) introduce a monotonicity axiom which leads to an answer in the affirmative: for all $x, y \in X$, $\{x, y\}$ offers strictly more opportunity freedom than each of the two sets $\{x\}$ and $\{y\}$. Sen (1993, 1988, 1992) condition the answer to what the decision maker actually prefers. If x is preference–wise better than y, then adding the alternative of doing x to the set $\{y\}$ enhances the extent of individual freedom. As argued in section 2, these axioms capture the substantive value of freedom, without paying attention to the effect of the addition of a new alternative on the agent's deliberative process.

Sugden (1998) makes a step further and refines the property as follows: the addition of an opportunity is autonomy freedom enhancing if, and only if, it makes a relevant choice, where relevance is evaluated in terms of the preference orderings of a reasonable person. If reasonableness is abandoned, as we have argued it should be, the addition of a new opportunity enlarges the chooser's extent of autonomy freedom if it makes a relevant choice in the light of the set of potential preference relations that that the decision maker is aware of. Hence, the 'irrelevance' of an opportunity can be determined by the limited set of available preference orderings; i.e., by his unawareness in the choice of a specific opportunity. This is the idea expressed by axioms AUA and ADA.

Each of the principles introduced so far, i.e., the indifference between no-freedom situations, the addition of relevant alternatives, the addition of irrelevant alternatives and the transitivity of the relation $\succeq$, seems reasonable and normatively appealing; however, as noted by Jones and Sugden (1982) and Sugden (1998), jointly considered they are logically inconsistent. This inconsistency is robust enough to extend to rankings defined over the space of opportunity situations as we now show by means of a mere extension of Jones and Sugden's proof to our framework.

Proposition 4.1 *No transitive binary relation $\succeq$ over $\mathcal{P}(X) \times \mathcal{P}(\Pi)$ exists that satisfies INF, AUA and ADA (Jones and Sugden 1982).*

Proof. Consider any $x, y \in X$ and suppose that xRy for all $R \in \Pi$. Then, by AUA,

$$\forall \Pi_i \in \mathcal{P}(\Pi), (\{x, y\}, \Pi_i) \succ (\{y\}, \Pi_i)$$

and, by ADA,

$$\forall \Pi_i \in \mathcal{P}(\Pi), (\{x, y\}, \Pi_i) \sim (\{x\}, \Pi_i)$$

hence, by transitivity,

$$\forall \Pi_i \in \mathcal{P}(\Pi), (\{x\}, \Pi_i) \succ (\{y\}, \Pi_i).$$

But, by INF,

$$\forall \Pi_i \in \mathcal{P}(\Pi), (\{x\}, \Pi_i) \sim (\{y\}, \Pi_i).$$

Q.E.D.

What is the root of this incompatibility? Consider the axiom AUA: it states that the addition of an opportunity, which is preferred to all the other available opportunities by all available preference relations, is freedom enhancing. Now suppose we were ranking opportunity situations on the basis of indirect utility, instead of autonomy freedom. Axiom AUA would then be satisfied; for, according to indirect utility, we should rank situations by looking at the most preferred alternative among those available, and only at that alternative. This observation unveils the fact that axiom AUA captures that aspect of opportunity freedom linked to the possibility of choosing what is valuable to us. But, as we have argued, there is at least one additional reason for valuing opportunity that one should consider: the procedural aspect, which attaches importance to the procedure of free decision *per se*. This aspect includes the feature of decisional autonomy and is clearly reflected in our axiom INF: if two opportunity situations lead to as many choice sets which are singletons, then there is no possibility for choice and the extent of autonomy freedom that they offer is nil.

A comprehensive assessment of freedom, Sen argues, should take note of both the opportunity and process aspects. In general there needs not to be a contradiction between the two: considering a very simple case, if the alternative x is preferred to y, we could have a ranking $\{x\} \succ \{y\}$, but also $\{x, y\} \succ \{x\} \succ \{y\}$, where clearly $\{x, y\} \succ \{x\}$ reflects the procedural value of having opportunity, whereas $\{x\} \succ \{y\}$ reflects the substantive value. However, in our case there is a logical inconsistency between axioms inspired by these two values of having opportunity. This happens, we believe, because axiom AUA gives too much support to the substantive value: for, according to property AUA, if there is an alternative x which is universally considered as better than another alternative y, then the addition of x to the set $\{y\}$ is freedom enhancing. And this applies even if the distance between x and y is as large as to make the choice problem trivial. Hence, this excludes any consideration of the autonomy as informed choice enjoyed in the process of choosing.

As a consequence, we reformulate axiom AUA. The idea is the following: in order to be freedom enhancing, a new alternative not only has to dominate the existing ones in terms of some preference ordering available to the individual—as in axiom AUA; it also should not be 'as good' as to make the choice problem trivial. Hence, the addition of x to the set $\{y\}$ is freedom enhancing if the following two conditions are satisfied: (*i*) x is preferred to y by some preference ordering available to the individual, which ensures that x is in the choice set elicited from the opportunity set $\{x, y\}$; (*ii*) yet, there exists another potential preference relation such that, if y belongs to the choice set of $\{x, y\}$, then it is not preference–wise dominated by x. We formulate this requirement in the next axiom.

Axiom 4.4 *Addition of relevant alternatives (ARA)*
$\forall (A, \Pi_i) \in P(X) \times P(\Pi), \forall x \in X \backslash A$, $[\exists R_h \in \Pi_i$ *such that* $x R_h y \ \forall y \in A$ *and,* $\forall s \in \max_i(A), \exists R_k \in \Pi_i$ *such that* $s R_k x] \Rightarrow (A \cup \{x\}, \Pi_i) \succ (A, \Pi_i)$.

An inspection of the argument used in the previous theorem readily shows that this axiom is compatible with the other properties introduced so far. Moreover, as we shall see, it leads to the characterization of a unique ranking of opportunity situations.

Our last property requires that, under certain circumstances, joining sets together does not alter their relative ranking in terms of autonomy freedom.

Axiom 4.5 *Composition (COM)*
$\forall A, B, C, D \in \mathcal{P}(X)$, $\forall \Pi_i, \Pi_j \in \mathcal{P}(\Pi)$, *such that* $max_i(A) \cap max_i(C) = max_j(B) \cap max_j(D) = \emptyset$, $[(A, \Pi_i) \succeq (B, \Pi_j)$ & $(C, \Pi_i) \succeq (D, \Pi_j)] \Rightarrow (A \cup C, \Pi_i) \succeq (B \cup D, \Pi_j)$.

Composition is intuitively related to the independence axiom originally introduced in Suppes (1987) and to the composition property introduced in Sen (1991). To see its significance, it may be helpful to start with a crude version of it and then explaining the successive refinements. Suppes' formulation is the following: if a set A is judged to give at least as much opportunity freedom as another set B, then that ranking will be unaffected by the addition to or subtraction from each of an alternative x not contained in either; i.e., given $x \notin A \cup B$, $A \succeq B$ if and only if $(A \cup \{x\}) \succeq (B \cup \{x\})$.

Now consider four sets, A, B, C and D, such that $A \succeq B$ and $C \succeq D$. A natural extension of the property above would be the following: $A \succeq B$ if and only if $(A \cup C) \succeq (B \cup D)$. However, a difficulty arises: suppose that A and C have many elements in common, while $B \cap D = \emptyset$. Then, adding D to B presumably increases an agent's opportunity freedom a lot, while adding C to A may not increase it much. Hence Sen's refinement: a ranking of two sets is unaffected by the addition of two other sets if both the pair of sets to be added have no common element: $A \succeq B$ is equivalent to $(A \cup C) \succeq (B \cup D)$ only if $A \cap C = B \cap D = \emptyset$. Our axiom relax Sen's condition by requiring that, for each pair of sets under consideration, the 'empty intersection' condition should refer only to relevant elements; i.e., the two sets to be added should not have maximal elements in common.

On the basis of these axioms we are able to prove the following proposition which establishes a unique ranking for opportunity situations.

Definition 4.1 *Autonomy as Informed Choice Ordering (AIC)*
For all $A, B \in \mathcal{P}(X)$, *for all* $\Pi_i, \Pi_j \in \mathcal{P}(\Pi)$,

$$(A, \Pi_i) \succeq_{AIC} (B, \Pi_j) \Leftrightarrow |max_i(A)| \geq |max_j(B)|.$$

Proposition 4.2 *$\succeq$ satisfies INF, ARA and COM if and only if* $\succeq = \succeq_{AIC}$

According to proposition 4.2, an individual i enjoys more autonomy freedom than another individual j if and only if the choice set that his preference relations elicit from his own opportunity set A has at least as many elements as the choice set that j can elicit by means of his own preference relations from B.

5 Autonomy and opportunity freedom

Before concluding we wish to draw the attention of the reader toward the relationship that exists between our ranking and some of the main results achieved so far in the literature. This should also provide some information about the generality of our approach. In particular, as the following remarks illustrate, the rule proposed in this paper has the nice property of generalizing two important results so far axiomatized in the literature.

Remark 5.1 *Suppose that the set of preference orderings* Π *satisfies a "richness" assumption, such that* $\forall A \in \mathcal{P}(X)$, $max(A) := \{x \in A : \neg\exists y \in A \text{ such that } yRx \text{ for some } R \in \Pi\} = A$. *Then, if* $\forall i \in N, \Pi_i = \Pi$, *all possible preference relations can be hold by the individuals, and the ranking established in theorem 4.2 coincides with the Simple Cardinality-based Ordering of Pattanaik and Xu (1990).*

Upon reflection, the richness assumption makes possible for every single available opportunity to forge a relevant choice in the comparison with any other opportunity. In this special case, any choice is an informed one and the autonomy ranking collapses to the simple cardinality of the available opportunities.

Alike their simple cardinality measure, Pattanaik and Xu's autonomy ranking based on Sugden's notion of reasonable preferences can be subsumed to by our rule, too. It suffices to restrict the set of preference relations the decision maker is aware of to that of the reasonable preference relations.

Remark 5.2 *If* $\forall i \in N, \Pi_i = \Pi^*$, *where* Π^* *stands for the set of reasonable preference profiles à la Sugden (1998) and Pattanaik and Xu (1998), then the ranking established in proposition 4.2 coincides with the rule characterized by Pattanaik and Xu (1998) (proposition 5.1).*

Finally, as the following remark illustrates, when the preference relation is given and linearity is satisfied, all opportunity situations should be ranked indifferently.

Remark 5.3 *If* $\forall i, \Pi_i$ *is a singleton and the preference relation is linear, then* $\forall A \in \mathcal{P}(X)$ *and* $\forall \Pi_i \in \mathcal{P}(\Pi)$, $\max_i(A) = \{x\}$ *and therefore* $\forall A, B \in \mathcal{P}(X)$ *and* $\forall \Pi_i, \Pi_j \in \mathcal{P}(\Pi), (A, \Pi_i) \sim (B, \Pi_j)$.

As the remarks above show, the formal framework we use and the result we obtain can be interpreted as a generalization of settings and results proposed so far in the literature. The generalization comes from the choice to consider opportunity situations, instead of opportunity sets, as the ‘objects’ to rank.

Though our approach carries a specific interpretation grounded on a particular view of autonomy freedom, however, the opportunity situation framework is potentially applicable outside the specific interpretation that we have chosen to adopt.

6 Conclusion

Measuring autonomy freedom has been so far accomplished by looking at a decision maker’s relevant opportunities when his admissible potential preferences are elicited on the basis of what is reasonable. In this paper we have illustrated the limits of this approach and argued in favor of awareness as an alternative criterion for eliciting the set of admissible potential preferences. On the basis of awareness, we have been able to construct a ranking based on information about all available opportunities and all preference relations the decision maker is aware of.

The major conceptual effort that we have proposed in this paper is that, in the assessment of autonomy, we should have an eye on the subjective circumstances in which an individual makes his choice. To this problem we have tried to give a solution. Though our approach has provided an environment where circumstances do play a role, further effort should be done along such a line of research which should also take into account the empirical applicability of our theoretical results.

Appendix: The proof of proposition 4.2

Before proving proposition 4.2, we state and prove the following lemma.

Lemma 6.1 *If $\succeq$ satisfies INF and COM, then, $\forall A \in \mathcal{P}(X), \forall \Pi_i \in \mathcal{P}(\Pi)$,*

$$(A, \Pi_i) \sim (max_i(A), \Pi_i).$$

Proof. If $\max_i(A) = A$, then the result clearly follows. If not, suppose $|max_i(A)| = g$ and let $max_i(A) = \{a_1, \ldots, a_g\}$ and $A - max_i(A) = \hat{A}$. Now, $max_i\left(\{a_1\} \cup \hat{A}\right) = max_i(\{a_1\}) = \{a_1\}$ and $max_i\left(\{a_2\} \cup \hat{A}\right) = max_i(\{a_2\}) = \{a_2\}$. Hence by INF,

$$\left(\{a_1\} \cup \hat{A}, \Pi_i\right) \sim (\{a_1\}, \Pi_i)$$

and

$$\left(\{a_2\} \cup \hat{A}, \Pi_i\right) \sim (\{a_2\}, \Pi_i)\,.$$

Clearly, $\{a_1\} \cap \{a_2\} = \emptyset$, $max_i\left(\{a_1\} \cup \{a_2\}\right) = \left(\{a_1\} \cup \{a_2\}\right)$, $\left(\{a_1\} \cup \hat{A}\right) \cap \left(\{a_2\} \cup \hat{A}\right) = \hat{A}$ and $\hat{A} \cap \left(max_i\left(\left(\{a_1\} \cup \hat{A}\right) \cup \left(\{a_2\} \cup \hat{A}\right)\right)\right) = \emptyset$. Hence we can apply axiom COM and obtain,

$$\left(\{a_1\} \cup \{a_2\} \cup \hat{A}, \Pi_i\right) \sim (\{a_1\} \cup \{a_2\}, \Pi_i)\,.$$

By considering successively $a_3, a_4, ..., a_g$, and applying INF and COM repeatedly, we finally obtain

$$\left(max_i\,(A) \cup \hat{A}, \Pi_i\right) \sim (max_i\,(A)\,, \Pi_i)$$

or

$$(A, \Pi_i) \sim (max_i\,(A)\,, \Pi_i)\,.$$

Q.E.D.

We are now in the position to prove proposition 4.2.

Proof. Necessity is straightforward. We therefore prove sufficiency. To start with, we show that

$$|max_i(A)| = |max_j(B)| \Rightarrow (A, \Pi_i) \sim (B, \Pi_j). \tag{1}$$

Suppose $|max_i(A)| = |max_j(B)| = g$. It follows that, $max_i(A) = \{a_1, \ldots, a_g\}$ and $max_j(B) = \{b_1, \ldots, b_g\}$. Using INF, $(\{a_1\}, \Pi_i) \sim (\{b_1\}, \Pi_j), (\{a_2\}, \Pi_i) \sim (\{b_2\}, \Pi_j)$ and $\{a_1\} \cap \{a_2\} = \emptyset$. Now, $max_i\{a_1, a_2\} = \{a_1, a_2\}$, so we can use axiom COM to yield:

$$(\{a_1, a_2\}, \Pi_i) \sim (\{b_1, b_2\}, \Pi_j).$$

By INF, $(\{a_3\}, \Pi_i) \sim (\{b_3\}, \Pi_j)$; by COM,

$$(\{a_1, a_2, a_3\}, \Pi_i) \sim (\{b_1, b_2, b_3\}, \Pi_j)$$

and so on. Finally we have:

$$(\{a_1, ..., a_g\}, \Pi_i) \sim (\{b_1, ..., b_g\}, \Pi_j),$$

i.e.,

$$(max_i(A), \Pi_i) \sim (max_j(B), \Pi_j). \tag{2}$$

Since $\succeq$ satisfies INF and COM, we can apply lemma 6.1 and obtain

$$(A, \Pi_i) \sim (max_i\,(A)\,, \Pi_i)\,. \tag{3}$$

and

$$(B, \Pi_j) \sim (max_i(B), \Pi_j). \tag{4}$$

Now, equations (2), (3), (4), and transitivity of $\succeq$ imply $(A, \Pi_i) \sim (B, \Pi_j)$.

Now we show that

$$|max_i(A)| > |max_j(B)| \Rightarrow (A, \Pi_i) \succ (B, \Pi_j).$$

Suppose $|max_i(A)| = g + t$ and $|max_j(B)| = g$. So, $max_i(A) = \{a_1, \ldots, a_{g+t}\}$ and $max_j(B) = \{b_1, \ldots, b_g\}$. Now, $max_i(\{a_1, ..., a_g\}) = \{a_1, ..., a_g\}$. Hence, by (1),

$$(\{a_1, ..., a_g\}, \Pi_i) \sim (B, \Pi_j) \tag{5}$$

Now, $max_i(\{a_1, ..., a_{g+1}\}) = \{a_1, ..., a_{g+1}\}$. By ARA,

$$(\{a_1, ..., a_{g+1}\}, \Pi_i) \succ (\{a_1, ..., a_g\}, \Pi_i)$$

and, by (5) and transitivity of $\succ$,

$$(\{a_1, ..., a_{g+1}\}, \Pi_i) \succ (B, \Pi_j).$$

By adding $a_{g+2}, \ldots, a_{g+t}$ successively, and by using ARA repeatedly, we have

$$(\{a_1, ..., a_{g+t}\}, \Pi_i) \succ (B, \Pi_j)$$

i.e.,

$$(max_i(A), \Pi_i) \succ (B, \Pi_j). \tag{6}$$

We know from Lemma 4.1 that

$$(A, \Pi_i) \sim (max_i (A), \Pi_i).$$

Clearly, $(A, \Pi_i) \sim (max_i(A), \Pi_i)$, (6) and transitivity of $\succeq$ imply $(A, \Pi_i) \succ (B, \Pi_j)$.

Q.E.D.

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